Use of Spherical Harmonic Deconvolution Methods to Compensate for Nonlinear Gradient Effects on MRI Images

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Spatial encoding in MR techniques is achieved by sampling the signal as a function of time in the presence of a magnetic field gradient. The gradients are assumed to generate a linear magnetic field gradient, and typical image reconstruction relies upon this approximation. However, high-speed gradients in the current generation of MRI scanners often sacrifice linearity for improvements in speed. Such nonlinearity results in distorted images. The problem is presented in terms of first principles, and a correction method based on a gradient field spherical harmonic expansion is proposed. In our case, the amount of distortion measured within a typical field of view (FOV) required for head imaging is sufficiently large that without the use of some distortion correction technique, the images would be of limited use for stereotaxy or longitudinal studies, where precise volumetric information is required. Magn Reson Med 52: 115–122, 2004. © 2004 Wiley-Liss, Inc.

Key words: MRI; gradients; distortion correction; NMR plotting probe; spherical harmonics

The quality of an MR image is dependent upon the accuracy with which the physical position is spatially encoded. Since MRI data are now used routinely for stereotaxy, longitudinal studies of atrophy, and functional studies, it is critical to ensure that images have no distortion or inhomogeneity. The principal machine-dependent sources of this inhomogeneity are eddy currents, gradient nonlinearity, and $B_0$ and $B_1$ inhomogeneity (14). In this study we present an analytical approach to calculating and removing the effects of nonlinear gradients only. We chose to focus on a gradient-only solution because of the recent interest in short-bore, high-speed gradients. Although peripheral nerve stimulation is a limiting feature of short rise times, such gradients have been found to be useful in high-speed echo-planar imaging (EPI) of the heart and diffusion tensor imaging (DTI) of the brain. To achieve short rise times and avoid peripheral nerve stimulation, designers have restricted the length of gradients and limited the number of turns in those gradients. Although these constraints are suitable for the implementation of pulse sequences with the desired speed, they have the undesired consequence of increased nonlinearity.

Nonlinear pulsed field gradients induce image distortions due to incorrect spatial encoding of the signal. If we assume that the field gradients are linear, it follows that $k$-space is sampled linearly, and thus the fast Fourier transform (FFT) is suitable for reconstruction. However, any deviation from linearity in the gradients results in nonlinear data sampling and subsequent errors in image spatial encoding. A nonlinear FT would allow these data to be correctly transformed to an image. Unfortunately, nonlinear FT greatly increases computation time by $N\log_2N$, relative to an FFT, which makes real-time image generation computationally prohibitive. Here we present a general analytical solution to correct image distortions induced by gradient nonlinearity. The method is applicable to any gradient configuration. It is robust and, more importantly, is based upon an approach whereby the FFT is maintained for image reconstruction.

The notion of mapping and correcting such MRI distortions is not new. Two early papers by Schad et al. (15,16) discussed the “pincushion” effects seen with 2D phantoms. Subsequent schemes include using a stereotaxic frame as a reference marker (11,14), comparing phantom images from CT and MRI (7,19), and imaging specific MRI phantoms (1,13,17,18) that typically consist of an array of tubes filled with a suitable contrast agent. Mizowaki et al. (12) alluded to an important problem with phantom studies: after they assessed the reproducibility of their own phantom-based correction, the authors concluded that such a correction may only be applicable in limited situations. Field mapping is another approach for determining distortion via induced phase shift (2,3,5). Other correction schemes have been proposed for particular problems, including a method for correcting distortions induced by the use of biplanar gradients (10). Regardless, these approaches are of limited use for discerning, modeling, or correcting distortion due to part of the MRI system in isolation (e.g., the gradients).

A recent paper (9) suggested a method for correcting both the intensity variations and the geometric distortions induced by nonlinear gradients, based on treating gradient coils as either an opposed Helmholtz pair for the $Z$-gradient or a Golay arrangement for the transverse gradients. However, this approach treats the nonlinear component of the gradient field as a constant for the intensity correction, and demonstrates correction of the nonlinear induced distortion by correcting a large phantom (a cube approximately 20 cm long). It is also assumed that for the chosen gradient geometry, only second-order terms are important. While this approach is useful for gradients with a significant length-to-diameter ratio ($>2$), it is of limited use for cur-
rent high-speed gradients with significant higher-order gradient field impurities.

In a recent patent application (8), it was suggested that spherical harmonics may be useful for developing a general robust method; however, no such method was described. Another patent application (20) proposed a method to reconstruct k-space based upon an assumed nonlinearity. This method involves the use of approximately \(10^{14}\) triple integrations, and thus is of limited practical value. Nevertheless, this patent application does strike at the heart of the problem: any method that is of practical value must use the FFT as its basis.

THEORY

In MRI, image \(r = (x, y, z)\) and \(k = (k_x, k_y, k_z)\) space are connected by an FT. During a pulse sequence, magnetic field gradients play the crucial role of generating k-space coverage. The signal from a single RF excitation of the whole sample in the presence of a set of three orthogonal gradients may be written as the FT

\[
s(k) = \int \rho(r) e^{-i2\pi k \cdot r} d^3 r,
\]

where \(s(k)\) is the signal in k-space, and \(\rho(r)\) is the spin density in image space. The reconstructed image, \(\hat{\rho}(r)\), is the inverse FT of the measured data, \(s_m(k)\)

\[
\hat{\rho}(r) = \frac{1}{(2\pi)^3} \int s_m(k) e^{i2\pi k \cdot r} d^3 k.
\]

The three implicitly time-dependent components of \(k\) are related to the respective gradient-component integrals

\[
k_x = \gamma \int G_x(r, t') dt', \quad k_y = \gamma \int G_y(r, t') dt', \quad k_z = \gamma \int G_z(r, t') dt'.
\]

Here, \(\gamma\) is a constant, and \((G_x, G_y, G_z)\) are the gradient components. The gradient field is a function (nonlinear) of the position vector \(r = (x, y, z)\). In standard MRI, k-space is sampled at a single time rate, under the assumption that linear gradient fields are applied. If the gradient field is nonlinear, there will be a geometric distortion of these images. Knowing the exact gradient field profiles is the key to solving this problem.

Obtaining an accurate description of the gradient field distribution is not a simple task. The most general approach is to expand the field using spherical harmonics as the basis function (6). The field \(B_{ij}^Z\) generated by a gradient field \((V = X, Y, Z)\) can be written in spherical coordinates as follows, where \(B_{(n,m)}^Z(r, \theta, \phi)\) is a spherical harmonics expansion of order \(n\) and degree \(m\) of each component of the gradient field, and has the form

\[
B_{(n,m)}^Z(r, \theta, \phi) = r^n a_{(n,m)}^Z \cos(m \phi) + b_{(n,m)}^Z \sin(m \phi)
\]

where \(a_{(n,m)}^Z\) and \(b_{(n,m)}^Z\) are constants, and \(r\) is the radial distance from the magnet isocenter. The associated Legendre functions are \(P_{m,n}(\cos \theta)\). With a finite number of terms, the summation of Eq. [4] is only an approximation of the true gradient field \(B_Z\). The use of spherical harmonics deconvolution to describe magnetic field impurities has been described previously (6). In general, a multipoles sampling of the diameter of the spherical volume (DSV) is performed to ensure oversampling and accurate estimation of the required gradient impurities. Magnets are usually mapped with up to 24 planes, yielding harmonics to the 3rd order. Depending on the gradient set and the design parameters employed, gradient impurities can extend to 7th order, requiring at least an eight-plane plot. Even gradient coils from the same manufacturer will have winding errors and thus variations from the predicted field, such that it would be inaccurate to use a theoretical field expansion. The optimum method is to measure the actual field strength produced by a particular gradient coil set at a finite number of points in the image space. From this sampling, a function can be constructed to accurately describe the field distribution.

Once knowledge of the gradient field \(B_Z\) is established, it can be defined as

\[
G_V(r) = \frac{d B_Z(r)}{d v} = \frac{d B_Z^X(v)}{d v} + \frac{d B_Z^Y(r)}{d v} = G_X + G_Y.
\]

Where the subscript \(v\) is used to denote a spatial dimension \((x, y, \text{or} z)\), \(B_Z\) is the total gradient field that is generated by a particular coil component, \(B_Z^X\) is the linear gradient field that has only the desired 1st-order harmonic \((v)\), and \(B_Z^Y\) is the nonlinear gradient field defined by the higher-order harmonics. For convenience, the subscript \(v\) will be dropped from \(B_Z\) in the following discussion. The gradient \(G_Z(r)\) contains the following linear (\(G_X\)) (Eq. [6]) and nonlinear (\(B_Z\)) components (Eq. [7]):

\[
G_Z^X = a_{X(1,1)}, \quad G_Y^X = b_{Y(1,1)}, \quad G_Z^Y = a_{Z(1,0)}
\]

By employing the notation

\[
\begin{align*}
\eta_X(x,y,z) &= \frac{B_X^2(r, \theta, \phi)}{G_X^2} \\
\eta_Y(x,y,z) &= \frac{B_Y^2(r, \theta, \phi)}{G_Y^2} \\
\eta_Z(x,y,z) &= \frac{B_Z^2(r, \theta, \phi)}{G_Z^2}
\end{align*}
\]
FIG. 1. The Bruker phantom used in this study. The grid markers are shown at 1, and can be seen throughout the phantom. The other markers are quality control for various pulse sequences.

Table 1
The Relative Value of the Coefficients of the Harmonic Impurities Obtained Using a 24-Plane Plot of a Sonata Gradient Set Interfaced to an Oxford Magnet Technology 4T whole-Body Magnet*

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Effect of 2mT/m GX</th>
<th>Effect of 2mT/m GZ</th>
<th>Harmonic</th>
<th>Effect of 2mT/m GX</th>
<th>Effect of 2mT/m GZ</th>
<th>Harmonic</th>
<th>Effect of 2mT/m GX</th>
<th>Effect of 2mT/m GZ</th>
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<td>b[4][2]</td>
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<td>-0.04</td>
<td>b[6][1]</td>
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<td>0.314</td>
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<td>b[4][3]</td>
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<td>-0.038</td>
<td>b[6][2]</td>
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<td>-0.002</td>
</tr>
<tr>
<td>b[1][1]</td>
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<td>-0.054</td>
<td>b[4][4]</td>
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<td>-0.468</td>
<td>b[6][3]</td>
<td>0.042</td>
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<td>a[2][0]</td>
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<td>a[5][0]</td>
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<td>-12.393</td>
<td>b[6][4]</td>
<td>-0.029</td>
<td>-0.016</td>
</tr>
<tr>
<td>a[2][1]</td>
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<td>0.009</td>
<td>a[5][1]</td>
<td>-10.861</td>
<td>0.022</td>
<td>b[6][5]</td>
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<tr>
<td>a[2][2]</td>
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<td>0.098</td>
<td>a[5][2]</td>
<td>0.035</td>
<td>0.055</td>
<td>b[6][6]</td>
<td>0.014</td>
<td>-0.002</td>
</tr>
<tr>
<td>b[2][1]</td>
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<td>b[2][2]</td>
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<td>a[5][4]</td>
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<td>-0.017</td>
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<tr>
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<td>0.007</td>
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<td>-0.044</td>
<td>0.003</td>
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<td>0.001</td>
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<td>a[4][3]</td>
<td>-0.028</td>
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<td>0.002</td>
<td>b[7][5]</td>
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<td>a[4][4]</td>
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<td>0.007</td>
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<td>b[4][1]</td>
<td>-0.072</td>
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<td>-0.004</td>
<td>b[7][7]</td>
<td>0.015</td>
<td>0.004</td>
</tr>
</tbody>
</table>

*The shimmed field is first mapped and the associated harmonics are subtracted from those generated by the gradients.
When the change in this term from subsequent iterations
can be measured by the
gradient.

The convergence of each iteration can be measured by the
geometric distortion:

where

\[
\begin{align*}
\theta &= \tan^{-1}\left(\frac{x}{y}\right), \\
\phi &= \tan^{-1}\left(\frac{y}{z}\right)
\end{align*}
\]

the mapping from the distorted to undistorted image space
can be written as

\[
\begin{align*}
x' &= x - \eta_x(x', y', z') \\
y' &= y - \eta_y(x', y', z') \\
z' &= z - \eta_z(x', y', z')
\end{align*}
\]

where for the first iteration, the initial values of \((x', y', z')\)
are \((x, y, z)\). Equation [10] can thus be solved iteratively for
\((x', y', z')\) via the following process:

\[
\begin{align*}
x'_i &= x - \eta_x(x_i, y_i, z_i) \\
y'_i &= y - \eta_y(x_i, y_i, z_i) \\
z'_i &= z - \eta_z(x_i, y_i, z_i)
\end{align*}
\]

\[
\begin{align*}
x'_i &= x - \eta_x(x'_{i-1}, y'_{i-1}, z'_{i-1}) \\
y'_i &= y - \eta_y(x'_{i-1}, y'_{i-1}, z'_{i-1}) \\
z'_i &= z - \eta_z(x'_{i-1}, y'_{i-1}, z'_{i-1})
\end{align*}
\]

The convergence of each iteration can be measured by the
term

\[
E(n) = \frac{\sqrt{(x'_n - x'_{n-1})^2 + (y'_n - y'_{n-1})^2 + (z'_n - z'_{n-1})^2}}{\sqrt{(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2}}
\]

When the change in this term from subsequent iterations
drops below a critical tolerance, iteration stops. In our
experience, a value of 0.01 is sufficient, and equates to 2.94
(0.0058) and 2.72 (0.0053) mean (standard deviation (SD))
iterations, respectively, for 5th- and 7th-order discrete
solutions.

This process does not correct for intensity variations
that arise from a nonlinear gradient field, nor does it yield
voxels with identical geometric dimensions. Clearly, for
this method to be useful in data analysis, the voxel
dimensions in 3D data must be geometrically equal (i.e., an image
is required at a regular set of points \((l_x, \Delta x, l_y, \Delta y, l_z, \Delta z')\)).
The following operations can be applied to correct the
geometric distortion:

\[
\begin{align*}
x_k &= m_x \Delta x' + \eta_x(m_x \Delta x', m_y \Delta y', m_z \Delta z') \\
y_k &= m_y \Delta y' + \eta_y(m_x \Delta x', m_y \Delta y', m_z \Delta z') \\
z_k &= m_z \Delta z' + \eta_z(m_x \Delta x', m_y \Delta y', m_z \Delta z')
\end{align*}
\]

From this point, a simple mass-preserving resampling of
image data using a discrete solution to the deformations
expressed in Eq. [13] is required as per Langlois et al. (9).
This method can be readily applied irrespective of the
particular pulse sequence and/or \(k\)-space trajectory.

**MATERIALS AND METHODS**

We performed a 24-plane spherical harmonic deconvolution
NMR plot on a set of high-speed Sonata gradients
manufactured by Siemens Aktiengesellschaft, Germany.
These gradients were interfaced to an Oxford Magnet
Technology 4T whole-body magnet. To measure the field
distribution of the gradients in isolation, we first plotted
the base passively-shimmed magnet, and then two more
plots in which a constant current was applied to the \(X\) and
\(Z\) gradients, respectively. By subtracting the base plot from
a plot with a gradient energized, one can determine the
effect of an individual gradient. For simplicity, we
assumed that the \(X\)- and \(Y\)-gradient distortions were
identical. To speed the correction, a discrete solution expressed
as a grid function of vectors over an area of interest
(±120 mm from the isocenter in \(x, y, \) and \(z\)) with a regular
spacing of 5 mm is generated from the set of spherical
harmonic coefficients. One generates each vector in the
volume by iterating toward the solution using the method
described in the previous section at each point in the grid, and storing the resultant delta values.

This set of deformation vectors can then be applied to any data set within its coverage of space. The time required to nonlinearly resample a 256³ image by this approach is ~20 min with average computing hardware (RISC 300 MHz), and the generation of the 5-mm 3D field only takes on the order of seconds. It is important to note that mass preservation resampling can be easily achieved with the above discrete technique via simple multiplication of the Jacobian determinant plus one of the deformation fields as part of the resampling. This type of resampling is often termed “mass-preservation resampling.”

We performed two imaging experiments: one using a phantom for reference to previous work, and one with a human head. The phantom was manufactured by Bruker Medical and is shown in Fig. 1. The phantom consisted of a cylinder (diameter = 18.5 cm, depth = 3 cm) with perfectly flat ends. Within the phantom were points marking known distances (20 mm for the main grid). Imaging was performed at 1.5T by means of a turbo spin-echo sequence with a 256 × 256 mm field of view (FOV; matrix = 512 × 512). The slice thickness was set at 5 mm, and TR/TE were set at 500/25 ms. The phantom was imaged in the YZ plane of the magnet.

The phantom was imaged a number of times at fixed distances along the X-, Y-, and Z-axes to give pseudo 3D coverage of the entire DSV of the gradients.

To demonstrate the effectiveness of the correction techniques on more conventional data, two 3D magnetization preparation rapid acquisition gradient echo (MPRAGE) T₁ images were acquired at 4T (TR = 2.5 s, TE = 3.93 ms, TI = 1100 ms, FOV = 240 mm, matrix = 256 × 256 × 256, TEM head coil) in a normal volunteer. The first was acquired with the head coil at the isocenter of the gradients, and the second was shifted 15 mm in the positive z-direction.

RESULTS

The plotted spherical harmonics to the 7th order for the gradients are shown in Table 1. The experimental magnetic field distributions for the X, Y, and Z gradients are shown in Fig. 2. It is clear from these isomagnetic plots that the gradients produce inhomogeneous magnetic fields, and it is likely that image distortion will result.

In order to visualize the results, it is useful to introduce the following parameter: For any gradient field, the quality of the linear gradient component can be measured by

\[ \Gamma (\mathbf{r}) = \frac{G^N (\mathbf{r})}{G^L}, \quad \| \mathbf{r} \| \leq r_{\text{ds}}. \]  

Here, \( r_{\text{ds}} \) is the radius of the DSV. The nonlinearity of the gradient field increases with the image volume to such an extent that the effective DSV in our case has a radius of <10 cm. (As shown below, the effective imaging space is best described by an elliptical volume.) Figure 3 demonstrates that the nonlinear behavior increases with the displacement from the isocenter.

FIG. 4. The phantom center is at the center of the imaging domain. The original (a) and corrected (b) images are shown.

FIG. 5. Spatial positions of the grid points measured in the YZ plane using the Bruker phantom. The open circles are measured positions, and the red dots indicate the corrected positions. Dimensions are in millimeters.
Figure 4a shows a 2D image of the phantom sliced in the XY plane with the center of the phantom placed at the magnet isocenter. Note that the grid markers are visible when this phantom is imaged. A red circle was superimposed on this image to demonstrate distortions.

We corrected the image using the harmonics to the 5th order listed in Table 1. The resultant image is shown in Fig. 4b. The background of the image was also corrected, illustrating that the nonlinear properties of the gradients increase for points further removed from the magnet isocenter.

We then corrected the image using the harmonics to the 5th order listed in Table 1. The resultant image is shown in Fig. 4b. The background of the image was also corrected, illustrating that the nonlinear properties of the gradients increase for points further removed from the magnet isocenter.

To examine the nonlinearity in a similar fashion to phantom studies that utilized control points, Fig. 5 shows the grid positions of the phantom when it was imaged in the YZ plane with its center 100 mm from the isocenter in Z. The open circles show the imaged position of the grid markers, and the red dots indicate their respective corrected positions.

To demonstrate the qualitative effects of the correction scheme on anatomic data, we corrected individual geometric distortions in both the 0-mm (isocenter) and 15-mm (plus 15 mm in the Z-plane) images using a discrete non-

FIG. 6. Difference maps for an uncorrected image and varying orders of correction (to orders 3, 5, and 7). Note the dramatic reduction in cortex anatomical differences in the corrected images as compared to the uncorrected image. An ROI of these changes is given in the right-most column. The left column shows the corresponding vector magnitude image, demonstrating the evolution of the inhomogeneity.
linear grid transform. To show the effect of the correction scheme with varying levels of complexity, we resampled the volumes using grid transforms generated from the spherical harmonic coefficients to the 3rd, 4th, 5th, 6th, and 7th orders. Mass preservation resampling was employed during the interpolation phase. We then registered the corrected image volumes using a rigid body transformation (4), and calculated a voxel difference map. Once both volumes are corrected and registered, the rigid structures (e.g., the cortex) should be identical. An axial slice from each of the corrected 3D volumes, and its associated difference images are shown in Fig. 6.

To quantitatively assess the correction scheme’s effectiveness on anatomical data, we calculated a normalized difference (z-score) between each 0-mm and 15-mm pair of volumes for each correction level. We chose a normalized difference measure as opposed to root mean squared error (RMSE) or cross correlation because even though the image pairs have similar intensity profiles, they may differ in mean and SD. Such differences often adversely affect simple measures of similarity. The normalized difference (z-score) is defined for two images X and Y as follows:

\[
\text{zscore} = \frac{1}{n} \sum_{i=0}^{n} \left( \frac{x_i - \bar{x}}{\bar{x}} \right) - \left( \frac{y_i - \bar{y}}{\bar{y}} \right).
\]

Where \(x_i\) is a sample from an image, \(\bar{x}\) is the image mean, and \(\bar{x}\) is the image SD. The resulting z-scores are shown in Fig. 7. The correction scheme shows incremental improvement when terms from the harmonic expansion are used to the 6th order. In this case, the addition of 7th-order terms slightly degraded the performance.

**DISCUSSION**

Image distortions induced by gradient nonlinearity are a serious problem for the generation of volumetric images by means of high-speed gradient sets. Although it might be possible to correct such distortions with the use of translation, warping, and twisting matrices, such approaches are empirical. In this work, we have demonstrated an image correction technique that quantifies gradient field impurities using spherical harmonics. It should be noted that it is assumed that any extra shimming required for image generation in an initially shimmed magnet is much smaller than the gross inhomogeneities in the gradient field.

As a qualification, we note that the image correction techniques outlined above are applicable for correcting image distortions arising from 3D acquisitions using phase encoding in the slice direction. This method is of limited utility for 2D data, since during the slice formation a nonlinear gradient field will influence which spins are excited, and whether these spins belong in the slice of interest. This problem must be resolved before a complete distortion correction method can be obtained.

Although the anatomical results are sufficient to demonstrate the utility of the method, they are not optimal. This is apparent in the degradation of correction performance that occurs when a 7th-order solution is used. There are two main reasons for this: First, the initial measurement of the gradient field was in X and Z only, and Y was assumed to be the same as X. Consequently, the data used in the correction scheme suffice only to a certain level. Second, although the metric used for image comparison (total z-score difference) is largely immune to image intensity scaling problems, it is not immune to B0 inhomogeneity. In this study, a spherical harmonic deconvolution of a Siemens gradient set was carried out at 4T. It is assumed that impurities are present to some extent in all gradients. Therefore, if this method is to be used routinely, gradients for a given scanner should be mapped and the corresponding variables stored in a table similar to Table 1. A discrete solution at the desired resolution can then be generated and applied on a routine basis to 3D images.

**REFERENCES**


